63

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## THE HIGH FREQUENCY LOW PRESSURE DISCHARGE

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## TABLE OF CONTENTS

## LIST OF FIGURES

# I. INTRODUCTION

Many studies both theoretical and experimental, have been conducted on the high frequency, low pressure discharge. The initial electron multiplication mechanism for this type of discharge is secondary electron emission from the end surfaces of the discharge vessel and a proper phase relation between the applied alternating electric field and the electron motion. The region of discharge excitation and maintenance depends upon this phase relation and the electric field strength which the electrons experience. The impedance of the space in which the discharge occurs therefore varies at certain excitation levels. Hence, such discharges possess an application to power activated microwave switching devices. The discharge produced by an internal electrode configuration has a promising property for high power applications. All the high frequency power absorbed by the device appears as heat in the internal metallic electrodes, which can be cooled with little difficulty under most conditions. This property is desirable when one contemplates the design of a high power device for high frequencies.

This paper is being written with the above applications in mind. But, we shall not discuss the design problem. The purpose of this paper is to consider theoretically the growth, maintenance, and extinction of the discharge phenomenon. In particular, we seek a better understanding of the role played by space charge effects during the history of the discharge.

With this introduction, we now state the conditions under which we wish to produce a discharge. The discharge will be discussed solely with reference to parallel plane geometry as shown in Figure 1. The end surfaces of the discharge region must have a secondary electron emission coefficient, $\delta$, greater than one in order for a discharge to develop. It is also essential to distinguish between two types of electrode placement with respect to the discharge vessel, as the behavior of the discharge depends upon the electrode placement. (See Figure 2) The metallic electrodes may be placed inside the discharge

vessel or outside the discharge vessel. In the former configuration, the end surfaces are conductors.[2] The latter configuration is often referred to as the electrodeless discharge in which the non-conducting, parallel plane walls of the discharge vessel form the end surfaces.

The parameters which determine growth, maintenance, and extinction of the discharge are the residual gas pressure, p, the frequency, f, of the applied electric field, E, and the dimensions of the vessel. The pressure, the mean free path of the electrons, $\lambda_e$, and the frequency, $\nu$, with which the electrons collide with the gas molecules are all inter-dependent quantities. The electric field is given by $E = E_o \sin \omega t = E_o \sin 2\pi ft = E_o \sin 2\frac{\pi c}{\lambda} t$, where $\lambda$ is the wavelength and c is the velocity of light. It is assumed that $\lambda$ is much greater than the dimensions of the vessel. This permits us to neglect the spatial dependence of the applied electric field. For our geometry, the distance between the plane surfaces and in the direction of the electric field E is d and the representative distance perpendicular to E and denoting the extent (width) of the vessel is r. The parameters are chosen such that $\lambda_e \gg d$, $\lambda_e \gg r$, $f \gg \nu$, and $\lambda \gg d$. Under this choice of initial conditions the electrons collide with the walls of the discharge vessel much more frequently than they collide with the residual gas molecules. The basic mechanism which produces breakdown will then be secondary electron emission from the end surfaces. For d and r of the order of centimeters, the pressure should be less than $10^{-2}$ mm Hg in order for the requirement $\lambda_e \gg d$ to be satisfied.

Let us now turn to a qualitative description of the discharge phenomenon. During the initial growth of the discharge, the space charge effects due to electrons are negligible, since the current density will be small. For multiplication to occur, two conditions must be satisfied simultaneously. Namely, the electron transit time should be $(l\pi/\omega)$, where $l = 1, 3, 5, \ldots$. The mode of shortest transit time, namely, $l = 1$, will be considered at this point for simplicity. In this way, the secondary electrons become primary electrons for the

next half cycle of the applied field to form another group of secondary electrons. The second condition is that the energy of the electrons when they arrive at the end surface must be such that $\delta \geq 1$. Also for multiplication to continue both the total number of electrons and their distribution in phase must be maintained. In order to allow for losses, $\delta$ must be greater than one in practice. The losses are mainly due to the angular distribution of the emitted electrons and to the reflection with almost undiminished speed of some of the primaries. The first loss tends to remove electrons from the discharge region and the second loss, which brings about excessive speed, may cause the electrons to become out of resonance with the field.

Under optimum conditions, where the electron arrival energy is a maximum, the electron must be in phase with the field. Thus an electron crossing the gap from one end surface should arrive at the other end surface and release $\delta$ secondary electrons just as the field passes through zero. The reversed electric field accelerates the $\delta$ secondary electrons back to the first end surface. The process repeats and after m traversals of the gap, there will be ideally $\delta$ m electrons in the gap. However, the effective multiplication is given by $\delta(1-a)^m$, where $\delta$ is the secondary electron emission coefficient necessary to overcome losses due to electron removal from the discharge region and where a represents the small fraction of electrons which are lost by falling out of phase. Since m is proportional to time, the electron multiplication increases exponentially with time until other effects such as space charge begin to limit and stabilize the process.

One should realize that multiplication does not require the optimum energy condition to be satisfied. The discharge can occur over a fairly broad region of field strengths and frequencies. For any one frequency, multiplication is possible in a bounded region between two values of field strength corresponding to too little or too much acceleration of the electrons to maintain the proper phase relations.

As the multiplication continues, space charge effects of the electrons become important and cannot be neglected any longer. In this and the next paragraph we assume that the effects of any positive charges which might have been created by ionization of the residual gas molecules are negligible. Electron space charge effects then begin to limit and stabilize the discharge. The electrons already emitted from an end surface tend to prevent the emission of other electrons. Also, during transit the Coulomb repulsion of the electrons tends to spread the electrons in all directions. Such repulsive forces may either remove electrons from the discharge region or cause those in the region to become out of synchronism with the applied field.

In the stable operating condition the fully developed discharge may be considered as a sheet of electrons moving back and forth between the end surfaces in synchronism with the applied field. The electron sheet is reconstituted by the secondary electrons at each impact with the end surfaces. The energy acquired during each transit is dissipated upon impact and the excess energy above that required to produce the secondary electrons then appears as heat in the end surfaces. This last characteristic is desirable for high power applications.

If the current density in the device is increased further by the application of a greater field strength, the discharge may or may not cease. The possibility of extinction depends upon the electrode placement and pressure. Firstly, both for the electrodeless discharge and for the internal electrode discharge, the disruptive effects of electron space charge will eventually extinguish the developed discharge when the pressure is less than about $10^{-4}$ mm Hg and when d is a few centimeters. Presumably, the positive ion accumulation is of no consequence under such conditions. Secondly, with $10^{-2} > p > 10^{-4}$ mm Hg, the developed electrodeless discharge can be extinguished. It is believed that in this range of pressures the positive ion accumulation influences the behavior of the electrons. The discharge mechanism is no longer the synchronous

secondary electron emission process described briefly above.

In order to explain the latter observations, we must not only examine the discharge mechanism when the electron space charge becomes appreciable, but also when the wall charge and the accumulated positive ion space charge, which results from ionization of the residual gas, are large enough to affect the electron motion. Even though the ionization rate is very low, positive ion space charges can become important. The positive ions move much more slowly than the electrons and the rate of de-ionization is very small. Hence, the ions can readily accumulate to form large space charges and to produce effects far out of proportion to the small number created per electron.

Many authors have presented a quantitative description of the initial breakdown mechanism which is applicable for both electrode configurations with d a few centimeters and $p < 10^{-2}$ mm Hg[1-6]. For completeness, we include the theory as Appendix A. At this point, the reader should familiarize himself with one of the above references or Appendix A.

## II. DESCRIPTION OF EXTINCTION

We now construct a model which can account for the extinction of the discharge when positive ion space charges and wall charges are not important. A self-consistent field approximation is developed. The effect of the shuttling electron sheet is replaced by an effective potential. The motion of an average representative electron of the sheet under the influence of this effective potential is then examined. Although this treatment is not as rigorous as a treatment based on the distribution function it does have the merit of suggesting an explanation of the discharge extinction.

We view the discharge as an electron sheet having an approximate width $2a \ll d$ and a uniform charge density $\rho$, which moves with a phase $\phi$ between two planes. The $x = 0$ plane is at zero potential while the $x = d$ plane has the applied alternating potential $V_0 \sin \omega t$. The effective potential in the region between the two planes is desired. Since $\lambda \gg d$, this boundary value problem can be solved for the electrostatic case at a given instant of time. The sinusoidally varying quantities are then included in the derived expressions in order to describe the motion of a representative electron.

The solution to the following boundary value problem is desired:

(Refer to Fig. 1).

$$\nabla^2 V = \begin{cases} 0, & 0 < x < (p-a) \quad : \quad I \\ \frac{\rho}{\epsilon}, & (p-a) < x < (p+a) \quad : \quad II \\ 0, & (p+a) < x < d \quad : \quad III \end{cases}$$

The boundary conditions are:

$$V(0) = 0, \quad V(d) = V_0 ;$$

$$V_I(p-a) = V_{II}(p-a), \quad V_{II}(p+a) = V_{III}(p+a); \text{ and}$$

$$\left.\frac{dV_I}{dx}\right|_{x=(p-a)} = \left.\frac{dV_{II}}{dx}\right|_{x=(p-a)} ; \quad \left.\frac{dV_{II}}{dx}\right|_{x=(p+a)} = \left.\frac{dV_{III}}{dx}\right|_{x=(p+a)}$$

The general solutions for the respective regions are:

$$V_I = bx + c,$$

$$V_{II} = ex^2 + fx + g, \text{ and}$$

$$V_{III} = hx + j$$

Substituting the above general solutions into the boundary conditions and then solving the resulting set of simultaneous equations, we obtain:

$$b = \frac{4\,e\,a(p-d) + V_o}{d}\ , \quad c = 0;$$

$$e = \frac{-\rho}{2\epsilon} = \frac{+nq}{2\epsilon}, \quad f = \frac{e\,[4\,a\,p - 2(p+a)d] + V_o}{d}\ , \quad g = (p-a)^2;$$

$$h = \frac{4\,eap + V_o}{d}\ , \quad j = -4\,eap,$$

where $\epsilon$ is the permitivity of the vacuum, n is the number of electrons per unit volume in the sheet, and q is the electronic charge. The potential between $x = 0$ and $x = d$ is then given by:

$$V_I = \frac{2\,nq\,a}{\epsilon\,d}\,(p-d)x + \frac{V_o\,x}{d},$$

$$V_{II} = \frac{nq}{2\epsilon}\,x^2 + \frac{nq}{\epsilon}\,\frac{[2ap - (p+a)\,d]\,x}{d} + \frac{nq}{2\epsilon}\,(p-a)^2 + \frac{V_o\,x}{d},$$

$$V_{III} = \frac{2nq}{\epsilon}\,a\,\frac{p(x-d)}{d} + \frac{V_o\,x}{d}$$

The time varying quantities are now included for completeness; that is, $V_o \sin \omega t$ replaces $V_o$ and $a + (d-2a) \sin(\omega t + \phi)$ replaces p. The effective electric field $E = -(dV/dx)$ which an individual electron experiences has the form:

$$E_I = -\frac{2\,nqa(p-a)}{\epsilon\,d} - \frac{V_o}{d}$$

$$E_{II} = -\frac{nqx}{\epsilon} - \frac{nq}{\epsilon}\left\{\frac{2\,ap - (p+a)d}{d}\right\} - \frac{V_o}{d}$$

$$E_{III} = -\frac{2\,nqap}{\epsilon\,d} - \frac{V_o}{d}$$

The motion of an electron acted on by this effective field is given by the equation $m(d^2x/dt^2) = q\,E$, where the initial conditions are that $x = 0$ and $(dx/dt) = v_1$ at $\omega t_1 = \phi_1$. We introduce the following relations:

$$\tau = \omega t, \quad z = (x/d), \quad \delta = (a/d),$$

$$\alpha = \frac{nq^2}{\omega^2 m\epsilon} \quad \text{and} \quad \beta = \frac{V_o\, q}{m\omega^2 d^2}$$

and then the equations of motion become:

$$\ddot{z}_I = -\beta \sin\tau - 2\alpha\delta\,[(\delta-1) + (1-2\delta)\sin(\tau+\phi)]$$

$$\ddot{z}_{II} = -\beta \sin\tau - \alpha z - \alpha\,[2\delta(\delta-1) - (1-2\delta)^2 \sin(\tau+\phi)]$$

$$\ddot{z}_{III} = -\beta \sin\tau - 2\alpha\delta\,[\delta + (1-2\delta)\sin(\tau+\phi)]$$

The above differential equations are solved by the method of undetermined coefficients. The results are:

$$z_I = -A\sin\tau - B\cos\tau + \frac{C\tau^2}{2} + i\,\tau + r,$$

$$z_{II} = S\sin\sqrt{\alpha}\,\tau + U\cos\sqrt{\alpha}\,\tau + \frac{1}{(a-1)}\,(D\sin\tau + E\cos\tau) + \frac{C}{\alpha},$$

$$z_{III} = -A\sin\tau - B\cos\tau + F\frac{\tau^2}{2} + v\,\tau + W,$$

where

$$A = -[\beta + 2\alpha\delta(1-2\delta)\cos\phi]$$

$$B = -2\alpha\delta(1-2\delta)\sin\phi$$

$$C = -2\alpha\delta(\delta-1)$$

$$D = +[\alpha(1-2\delta)^2\cos\phi - \beta]$$

$$E = +\alpha(1-2\delta)^2\sin\phi$$

$$F = -2\alpha\delta^2$$

The constants I, S, U, v, and W are determined by the following conditions:

$$Z_I(\phi_1) = 0, \quad [d Z_I(\phi_1)/d\tau] = \gamma = (v_1/\omega d),$$

and z and $(dz/d\tau)$ are to be continuous across the planes $x = (p-a)$ and $x = (p+a)$, if the parameters are such that the electron can move from one region to another.

The quantity $v_1$ is the emission velocity of a secondary electron leaving the plane $z = 0$ at the time $\phi_1$. If the treatment were to be made closer to reality, it would be necessary to take into account the emission velocity distribution. However, assuming all secondaries to be emitted with the same velocity $v_1$ is not a serious defect when one considers that our electron sheet of uniform density $\rho$ only crudely approximates the physical system.

The actual determination of the above constants is a formidable task since one has to examine different cases; namely, whether or not the electron remains in a given region. We shall not attempt this here, but shall merely indicate in principle the approach one uses to describe the discharge. The equations of motion must satisfy four requirements if the discharge is to be sustained. Firstly, when $z = 1$, the elapsed transit time should be $\ell\pi$ or $\tau_1 = \phi_1 + \ell\pi$, where $\ell = 1, 3, 5, \ldots$. Secondly, the presence of the walls must be taken into account. This means that for $\phi_1 \leq \phi \leq \phi_1 + \ell\pi$, z must be in the interval $0 \leq z \leq 1$. Thirdly, the phase must be stable, that is, if the voltage is such that at some phase $\phi_1$ the electrons have transit angles $\ell\pi$, then electrons having slightly different initial phases $\phi_1 + d\phi_1$ should eventually converge to phases of $\phi_1$. If this were not the case, the discharge would not be stable. Fourthly, it is essential that an electron arriving at an end surface must have gained sufficient energy during the transit to create, on the average, at least one secondary electron. When these four conditions are taken into account, it is possible to estimate over what ranges of $V_o$ for a given $\omega$ a discharge can be maintained.

Even though a quantitative description of the developed discharge is not

readily avialable from this model, the model does however suggest a mechanism which accounts for the extinction of a discharge when the pressure is such that any residual molecules or ions have no appreciable effect on the electron motion. The motion of an electron in region II is the superposition of two characteristic motions; namely, one component having the frequency $\sqrt{\alpha}\,\omega$ and the other component having the frequency $\omega$. The amplitude of the $\omega$ component is proportional to $(\alpha/\alpha-1)$. This model then predicts a critical condition. As $\alpha$ approaches unity, the amplitude of electron motion increases without limit. Hence, the model indicates that whenever $\alpha \approx 1$, the electrons can no longer remain together in a sheet with a $\ll d$ and with the correct phase $\phi$. Even though this infinite amplitude prediction is a result of the simplifying assumptions made in formulating the mathematics of the problem, it does mean that the previous theory is inadequate for $\alpha \approx 1$ and that other modes of electron motion, which are not compatible with the maintenance of a phase dependent discharge, become excited.

The quantity $\alpha$ is given by:

$$(nq^2/\omega^2 mc) = (\omega_p/\omega)^2 = \alpha,$$

where $\omega_p$ is the plasma frequency. If $\alpha$ exceeds unity, the applied electric field only partially penetrates the region. The above phenomena then suggest an explanation of the discharge extinction. The frequency $\omega$ is considered to be fixed. Now, as the applied electric field increases, the electron density n also increases in order for the device to handle the greater power requirement. Then for some value of E, n can be such that $\alpha \approx 1$. Other modes of electron excitation which cause the sheet to become unstable then occur. These modes destroy the region of greater charge density moving in synchronism with the applied field and thereby bring about the cessation of the discharge.

# III. DISCHARGE STAGES

The discharge mechanism becomes more complicated when the wall charge and the accumulated positive ion space charge are sufficient to affect the electron motion. G. Francis has considered this problem and the following qualitative discussion is based upon his presentation[1].

Three different physical processes may occur during the history of a discharge. The first stage of discharge growth, which has already been mentioned, is governed by secondary electron emission at the end surfaces and increases exponentially with time. The second stage of discharge growth occurs when the positive ion space charge becomes sufficient to influence the electron motion. This second stage does not always necessarily have to exist. At very low pressures ($< 10^{-4}$ mm Hg) the self repulsion of the electrons becomes a dominant factor before a large number of ions have been produced.. During every transit, each electron has a certain probability of ionizing a gas molecule. This probability depends on the pressure, the nature of the gas, and the electron energy. The probability is greatest when the electron energy is greatest. The resulting distribution of ions in space is found to have a maximum at the center of the discharge region. The concentration of ions at the center is of the order two times that at the walls. Hence, for simplicity in calculations, one can assume a uniform distribution. These ions can be regarded as stationary since the electron sheet makes many traversals during the time required for an ion to move a distance of the order d. Therefore, a positive ion space charge accumulates in the center of the discharge region and attracts individual electrons towards this center region. The result is that the speed of the electron leaving an end surface increases and hence the ionization rate increases; while the electron arrival speed at the opposite end surface decreases and hence the secondary emission decreases. During this stage, both ionization and secondary emission are of equal importance in the production of new electrons.

Finally, the electrons originally produced by secondary emission are lost to the end surfaces whenever their arrival speed is such that δ is less than one and are replaced by electrons produced by ionization. The latter electrons then execute forced oscillation $\pi/2$ radians out of phase with the applied field about the central positive space charge, as an electron in free space does. The electron sheet now moves with an amplitude somewhat less than d. The sheet width increases slowly by self repulsion and electrons are deposited on the end surfaces at the same rate as they are produced by ionization. This third stage is possible only when the pressure is greater than about $10^{-4}$ mm Hg and when the end surfaces are of such a nature that a strong negative wall charge can develop.

At pressures less than about $10^{-4}$ mm Hg, the ion space charge density is not enough to control the electron oscillation. The above theory then predicts why an increasing field will extinguish the internal electrode discharge but will not extinguish the electrodeless discharge for the pressure range $10^{-4}$ mm Hg to $10^{-2}$ mm Hg. The strong negative end surface charge that develops in the electrodeless discharge aids the ion space charge in decreasing the amplitude oscillation of the electron sheet. This prevents the further loss of electrons produced by ionization to the end surfaces. But, electrons swept onto the metal electrodes of the internal configuration do not give rise to strong fields. The electron amplitude oscillation is thereby not diminished sufficiently to prevent the further electron loss. Hence, all electrons will eventually be removed from the region and the developed internal electrode discharge will cease.

## IV. POSITIVE ION DISTRIBUTION FOR THE STABLE DISCHARGE

It is useful to examine the potential distribution which arises from the positive ion space charge and to derive a relation between this potential and the electron sheet density. If a discharge is to be stabilized by the positive ion space charge, the space charge should not vary with time. For simplicity, we assume that the positive space charge density $\rho^+$ is uniform in the region $|x| \leq (d/2)$ and that the positive ions are being created at the constant rate $\nu_i n$, where $\nu_i$ is the number of positive ions created per second per electron. The justification of this last assumption is that the electron sheet moves orders of magnitude faster than the ions move. The only loss process for the case at hand is the loss of ions to the vessel walls. Any ion produced in the discharge region will eventually move toward the walls due to the ion self-repulsion. Hence, we seek the potential distribution which satisfies the condition that the number of ions created equals the number lost to the walls.

Referring to Fig. 3, we see that stability imposes the condition that

$$A\, dJ = q\, \nu_i\, n\, A\, ds, \tag{1}$$

where A is the area of the parallel end surfaces, and where $dJ$ is the increment of ion current density at $x = (d/2)$ which originated between $s$ and $s + ds$. If $v$ is the velocity of the current $dJ$ at $x = (d/2)$ just before it is lost to the walls, then

$$v\, d\rho^+ = dJ \tag{2}$$

where $d\rho^+$ is the charge density at $x = (d/2)$ composed of charges which originated between $s$ and $s + ds$. We also have

$$v = \left[\frac{2q}{m^+}\,[V(s) - V(d/2)]\right]^{1/2}, \tag{3}$$

where $V(x)$ is the potential due to ions at $x$. Poisson's equation gives us

$$\frac{d^2V}{dx^2} = -\frac{\rho^+}{\epsilon}, \tag{4}$$

since we are neglecting the electron space charge in this treatment. Combining the Eqs. (1), (2), (3), and (4) yields

$$\rho^+ = \int d\rho^+ = -\epsilon \frac{\partial^2 V}{\partial x^2} = \int_0^{\frac{d}{2}} \frac{q\, v_i\, n\, ds}{\left[\frac{2q}{m^+}\,[V(s) - V(d/2)]\right]}$$

Since $\rho^+$ is a constant, the potential is assumed to be of the form

$$V = -B_1 x^2$$

This gives us

$$\rho^+ = 2 B_1 \epsilon = \frac{q v_i n}{\left\{\frac{2 q B_1}{m^+}\right\}^{1/2}} \sin^{-1}\left(\frac{2s}{d}\right) \Bigg|_0^{d/2}$$

or

$$B_1 = \left[\frac{q v_i n \pi}{4\epsilon \left(\frac{2q}{m^+}\right)^{1/2}}\right]^{2/3} \qquad (5)$$

Relation (5) represents a condition which must be fulfilled if positive space charge accumulation is to be the mechanism by which the discharge is brought to an equilibrium state for pressures between $10^{-4}$ mm Hg and $10^{-2}$ mm Hg. The ion density should vary approximately as the two-thirds power of the electron density in the sheet. However, one must keep in mind the unsophisticated manner in which the relation was derived. At best, it will probably give only order of magnitude agreement with experiment. Nevertheless, it does predict that the discharge might be extinguished whenever other processes which prevent fulfillment of the relation exist.

## V. NUMERICAL CALCULATIONS

In this section, we present order of magnitude calculations to show the plausibility of the statements and assumptions which appear in the previous sections.

Let the distance, d, be one centimeter. Then the requirements $\lambda >> d$ and the desirability of operating in the proper part of the breakdown region, which is obtained from $\omega$ versus $(V_o/d)$ plots, dictate that $\lambda$ should be of the order 200d. We shall assume $\lambda = 300$ cm or $f = \frac{\omega}{2\pi} = 10^8$ cps. The discussion will be limited to the $\ell = 1$ mode for which the half period is $5 \times 10^{-7}$ sec. The average velocity of the electron sheet is (1 cm/$5 \times 10^{-7}$ sec) or $2 \times 10^8$ cm/sec. This corresponds to an average energy of 11ev. The secondaries are emitted with energies of the order 5 ev to 10 ev and the impact energies which are necessary in order that $\delta$ be greater than unity are of the order of 100 ev for most substances.

The mean free path of an 100 ev electron in air is $\lambda_e \approx (1/23p)$cm where p is given in mm Hg. Also, for a 9 ev or 10 ev electron the mean free path is $\lambda_e \approx (1/40p)$cm. Hence, for pressures less than $10^{-2}$ mm Hg the condition $\lambda_e >> d$ is satisfied. There are $3.54 \times 10^{16}$ (molecules/cm$^3$) for each mm Hg of residual gas pressure at 0°C. Therefore the number of residual gas molecules per cubic centimeter at a pressure p and temperature T is $3.54 \times 10^{16} \left( \frac{273}{T} \right) p$.

The plasma frequency is $f_p = (\omega_p/2\pi) = 8.980 \times 10^3 \sqrt{n}$ cps. The condition that $\alpha \approx 1$ corresponds to an electron density of the order $10^8$ (electrons/cm$^3$). Such electron densities are easily obtainable in most discharges.

The positive ions have thermal energies of the order $kT \approx 3.76 \times 10^{-14}$ ergs or $2.35 \times 10^{-2}$ ev. This energy corresponds to a velocity of about $(2.12/Z^{1/2}) \times 10^5$ cm/sec where Z is the molecular weight. Hence, the ions indeed move much more slowly than the electron sheet moves.

In order to evaluate $B_1$ we assume that the residual gas is air. We have $v_i = p\, P_i\, v$ where p is the pressure in mm Hg. $P_i$ is the number of ions produced by one electron per centimeter of path at a pressure of one mm Hg and where v is the electron velocity in (cm/sec). From data[5], we see that an 100 ev electron yields a $p_i \approx 7$ for $N_2$, $O_2$, NO, and $C_O$. Air is composed mainly of $N_2$ and $O_2$ and has an effective molecular weight of 29. Substituting these numbers into Eq. (5) and being particularly cautious in the use of units, we find

$$B_1 = .81(pn)^{2/3} \text{ (volts/cm}^3\text{)},$$

where n is the number of electrons in the sheet per cubic centimeter and where p is the pressure in mm Hg. When $p = 10^{-3}$ mm Hg and $n = 10^6$ (electron/cm$^3$), the maximum potential $|V| = |B_1 d^2|$ due to positive ions is 81 volts. Indeed, one certainly expects an 81 volt potential to influence the motion of 9 ev to 100 ev electrons.

From this value of $B_1$ we can state the number of positive ions per cubic centimeter $n^+$, namely,

$$n^+ = \frac{\rho^+}{q} = \frac{2B_1 \epsilon}{q} = 8.93 \times 10^5 (pn)^{2/3} \text{ (ions/cm}^3\text{.)} \qquad (6)$$

The ratio $(n^+/n) = 8.93 \times 10^5 (p/n^{1/2})^{2/3}$.

When $p = 10^{-3}$ mm Hg and $n = 10^6$ electron/cm$^3$, the ratio $(n^+/n) = 89.3$. Therefore, we see that the positive ion density can be larger than the electron sheet for the above p and n values. Also, observe that there are about $10^{13}$ molecules per cubic centimeter of residual gas. Hence, only an extremely small fraction of the gas is ionized. But this small fraction

must be considered when investigating the discharge process.

The pressure at which the positive ion accumulation begins to influence appreciably the electron motion will now be calculated. The growth of the discharge may be checked by either the self-repulsion and dephasing of the electrons or by the positive ion space charge. The pressure and the applied frequency are crucial quantities in determining which of these two processes occurs first. The treatment presented in section II ignores the phase distribution and becomes invalid when the self-repulsion of the electron layer causes it to spread along the length of the vessel. Then motion in this case resembles a plasma oscillation. The condition $\alpha \approx 1$ gives us the maximum possible n which will sustain a discharge. However, the dephasing properties due to the distribution in phase $\phi$ most likely occures before $\alpha \approx 1$. Let the value of $\alpha$ be $\mu$, where $\mu < 1$ when the discharge ceases due to self-repulsion and dephasing of the electrons in the shuttling sheet. Then the maximum permissible n before extinction takes place due to the first process is given by

$$n < \left(\frac{m\omega^2\epsilon}{q^2}\right)\mu = \frac{f^2\mu}{(8.98 \times 10^3)^2} \text{ (electrons/cm}^3\text{)} \qquad (7)$$

From Figures 8 and 9 of reference 1, we have that the positive ions begin to affect the motion of the electrons when $(k/f)^2 \approx .2$, where $k^2 = n^+q^2/m\epsilon$ is the natural frequency of oscillation of the electron sheet under the influence of the ions alone. The quantity m is the electron mass. This is the point where the natural frequency k of the electrons oscillating in a uniform positive space charge becomes comparable with the applied frequency f. Combining the above condition with the stability relation (6), we determine the desired pressure for a given

frequency f above which ions are important. We write

$$k = 8.98 \times 10^3 \ n^+ \text{ cps},$$

and then have

$$k^2 = -2 f^2,$$

or when

$$n^+ = \frac{.2 f^2}{(8.98 \times 10^3)^2} \ (\text{ions/cm}^3)$$

the ions influence the electronic motion. We substitute this into the stability relation (6) and solve for pn. This gives us

$$pn = 1.47 \times 10^{-22} f^3.$$

Using the inequality (7), we obtain the pressure above which positive ions must be considered in the discharge mechanism; namely, whenever

$$p = \frac{1.47 \times 10^{-22} f^3}{n} > \frac{1.19 \times 10^{-14} f}{\mu} \qquad (8)$$

This inequality (8) indicates that experiments performed in such a manner that ($p/\lambda$) and pd) have the same respective values should yield similar results. From inequality (8), the critical pressure for transition from a self-repulsion checked discharge to a positive space charge checked discharge is directly proportional to $f = (c/\lambda)$ and the pressure is also

proportional to $(1/\lambda_e)$. Now, in order for the discharge to occur at all we must have $\lambda_e \gg d$ and we thereby arbitrarily state that $\lambda_e$ (large number) d. Therefore, p is also related to d through the mean free path condition. Hence, discharges with the same $(p/\lambda)$ and (p d) will behave similarly.

## VI. CONCLUSION

The high frequency, low pressure discharge of the type discussed in this paper is not readily amenable to being used as a high power microwave switching device. Firstly, we note in Section II and V that the electron density of the sheet is limited by the condition $\alpha < 1$ under all operation conditions. This condition restricts n to be less than $(m \omega^2 \epsilon/q^2)$. For frequencies of the order $10^8$ c.p.s. this means that n must be less than $10^8$ (electrons/$cm^3$). High power applications require electron densities which are generally greater than $10^{10}$ (electrons/$cm^3$). Stability conditions also prohibit the presence of such high electron densities.

The starting of the discharge depends on a chance electron being at the right place in a suitable phase $\phi$ with the applied field. Due to these requirements, the discharge is not likely to begin at the same instant of time after the application of each transmission pulse. These starting time fluctuations are reduced by providing a source of electrons. This is accomplished by illuminating the vessel with ultraviolet light or by reducing the time interval between transmission pulses, so that a few electrons from the previous discharge are still present to initiate the next one.

The time required for the discharge to develop and pass through the various stages of growth depends upon the applied field amplitude, the phase $\phi$, and the number of initial electrons having the phase $\phi$ and is thereby not readily controlled by external means. The usual development time is of the order of tens of r-f cycles.

Another difficulty is brought to mind when one examines the excitation levels of the discharge; especially, in those applications where the

amplitude of the applied signal may increase to such a value with time that the discharge parameters $V_o$ and $\omega$ pass through the permissible discharge region into the extinction region. Extinction of the discharge in a branched duplexer during the final portions of the transmission pulse would severely damage the receiver, would reflect energy back to the transmitter and thereby produce arcing, and would deliver very little energy to the antenna. All the above effects are undesirable.

The recovery time for the discharge is very short, or the order of a few r-f cycles, when compared with the recovery time for a gaseous discharge. Any remaining electrons in the gap after the applied signal ceases will be quickly lost to the walls. This discharge device should also have a very long life, compared to the conventional gas device. Since operating conditions can be made so that essentially no ions are present, the problem of gas clean up and sputtering does not plague the designer. Finally, the cooling of the device can be easily accomplished for only the electrodes are heated in the internal electrode discharge.

# APPENDIX A

## THEORY FOR INITIAL DISCHARGE BREAKDOWN

Consider two parallel opposed surfaces which are a distance d apart and between which an effective field, $(V_o/d) \sin \omega t$, appears.[7-10] The equation of motion of a representative electron under the influence of this field is.

$$m \frac{d^2x}{dt^2} = \frac{q V_o}{d} \sin \omega t. \tag{A1}$$

When we introduce the parameters

$$\tau = \omega t, \; Z = (x/d), \text{ and } \beta = (V_o \, q/m \, \omega^2 \, d^2),$$

Eq. (A1) becomes

$$\frac{d^2Z}{d\tau^2} = \beta \sin \tau. \tag{A2}$$

The boundary conditions are that at $\tau = \tau_o$

$$\frac{dZ}{d\tau} = \frac{v_1}{\omega d} = \gamma \quad \text{and} \quad Z = 0,$$

where $v_1$ is the emission velocity of a secondary electron leaving the plane $Z = 0$ at time $\tau = \tau_o$. The desired solution of Eq. (A2) is then given by

$$\frac{dZ}{d\tau} = \beta (\cos \tau_o - \cos \tau) + \gamma \tag{A3}$$

and

$$Z = \beta \left[ (\tau-\tau_o) \cos \tau_o - (\sin \tau - \sin \tau_o) \right] + \gamma(\tau-\tau_o), \qquad (A4)$$

for

$$\tau_o \leq \tau \leq \tau_o + \ell\pi.$$

The distribution of emission velocities will be neglected.

We now apply the discharge breakdown conditions. When Z = 1, the transit time should be $\ell\pi$ or $\tau_1=\tau_o + \ell\pi$ where $\ell$ = 1, 3, 5, ... Solving Eq. (A4) for $\beta$, we obtain

$$\beta = \frac{1 + \ell\pi\gamma}{\pi \cos \tau_o + 2 \sin \tau_o}, \qquad (A5)$$

as the first kinematic condition for breakdown to occur. Eq. (A5) represents a necessary, but not a sufficient, condition.

The next condition is that when $\tau_o \leq \tau \leq \ell\pi + \tau_o$, Z must be in the interval $0 \leq Z \leq 1$. The condition $Z \geq 0$ can be examined by considering the limiting case of $(dZ/d\tau) \geq 0$ at Z = 0. This will occur at some value $\tau = \tau_2$. We then have

$$0 = \beta (\cos \tau_o - \cos \tau_2) + \gamma,$$

and

$$0 = \beta (\tau_2-\tau_o) \cos \tau_o - (\sin \tau_2-\sin \tau_o) + \gamma (\tau_2 - \tau_o).$$

In principle, $\tau_2$ can be eliminated from these equations to yield a relation between $(\gamma/\beta)$ and $\tau_o$. The quantity $\tau_o$ that appears in the resulting expression is the maximum negative phase angle that will allow an electron to leave the surface Z = 1 against the retarding electric field for a given $(\gamma/\beta)$. The elimination can be done graphically.(6)

The condition that $Z \leq 1$ is equivalent to $\frac{dZ}{d\tau} > 0$ at $\tau_1 = \tau_o + \pi$ and from Eq. (A3) we obtain

$$2 \cos \tau_o > - (\gamma/\beta). \tag{A6}$$

If an electron leaves at time $\tau_o$ and arrives at time $\tau_1 = \tau_o + \ell\pi$ and if an electron starting at $\tau_s = \tau_o + \tau_3$ arrives at time $\tau_a = \tau_o + \ell\pi + \tau_4$, then phase stability (i.e., eventual convergence of the electron phase to $\tau_o$) requires that $/\tau_4/ < / \tau_3 /$ or

$$/\tau_a - \tau_1/ < / \tau_s - \tau_o /.$$

Therefore, stability imposes the requirement that

$$/ \frac{d\tau_1}{d\tau_o} / < / \text{ at } \tau_1 = \tau_o + \ell\pi.$$

From Eq. (A4) we have

$$1 = \beta [(\tau_1 - \tau_o) \cos \tau_o - (\sin \tau_1 - \sin \tau_o)] + \gamma(\tau_1 - \tau_o).$$

Differentiating with respect to $\tau_o$ and then setting $\tau_1 = \tau_o + \ell\pi$, we find

$$\frac{d\tau_1}{d\tau_o} = \frac{\ell\pi\,\beta \sin \tau_o + \gamma}{\gamma + 2\beta \cos \tau_o} .$$

The stability condition that $\left|d\tau_1/d\tau_o\right| < 1$ then yields

$$\left|\ell\pi\,\beta \sin \tau_o\right| < \left|2\,\beta \cos \tau_o\right|,$$

or

$$\left|\tan \tau_o\right| < \frac{2}{\ell\pi} . \tag{A7}$$

Finally, the impact energy $\Sigma_i$ must be such that $\delta$ is greater than unity. This gives us that $\Sigma_o < \Sigma_i < \Sigma_1$ where $\delta > 1$ for the interval $\Sigma_o < \Sigma < \Sigma_1$. The impact energy is given by

$$q\,V_i = \frac{1}{2}\,m\,v_i^2 = \frac{q\,V_o}{2\beta}\,(2\beta \cos \tau_o + \gamma)^2,$$

where $\beta$ satisfies Eq. (A5). It is useful to normalize energies to the energy $q\,V_o$. We write

$$\frac{V_e}{V_o} = \frac{(1/2)\,m\,v_1^2}{q\,V_o} = \frac{\gamma^2}{2\beta},$$

and

$$\frac{V_i}{V_e} = \left(\frac{2\beta}{\gamma} \cos \tau_o + 1\right)^2. \tag{A8}$$

From Eq. (A5) we have

$$\gamma = \frac{1-\beta\,(\ell\pi \cos \tau_o + 2 \sin \tau_o)}{\ell\pi}\,, \tag{A9}$$

and upon substituting Eq. (A9) into Eq. (A8) we see that

$$\frac{V_i}{V_e} = \left[\frac{2\,\beta \cos \tau_o}{\frac{1}{\ell\pi} - \left\{\cos \tau_o + \frac{2}{\ell\pi} \sin \tau_o\right\}\beta} + 1\right]^2$$

The condition $\delta > 1$ then yields the final restriction on $\beta$, namely,

$$\frac{\Sigma_o}{V_e} < \frac{V_i}{V_e} < \frac{\Sigma_i}{V_e}\,. \tag{A10}$$

Eqs. (A5), (A6), (A7), and (A10) are the conditions which $\beta$ must fulfill. Hence, the values of $\beta$ and $\tau_o$ over which a discharge should breakdown may be obtained theoretically from this simplified model.[1,5,6]

# APPENDIX B

In this appendix we indicate the formalism of a more sophisticated approach to the discharge phenomena. A method for obtaining the gross electrical properties is outlined.

The electron distribution function $f(\underline{r}, \underline{v}, t)$ satisfies Boltzmann's equation; namely,

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \underline{\nabla}_s f + \underline{a} \cdot \underline{\nabla}_v f = \Sigma_j C(f_1 F_j).$$

The quantity $f(\underline{r}, \underline{v}, t)\, d\underline{r}\, d\underline{v}$ is the probable number of electrons in volume $d\underline{r}$ about $\underline{r}$ having velocities in the range $\underline{v}$ and $\underline{v} + d\underline{v}$ at time $\tau$. The spatial gradient is $\underline{\nabla}_s$ and the velocity gradient is $\underline{\nabla}_v$. The quantity $\underline{a}(t)$ is the acceleration due to external forces. For our problem we assume $\underline{a}$ arises from the Lorentz force of a microwave electric field, that is,

$$\underline{a}(t) = (q/m)\, \underline{E} \sin \omega t.$$

The rf magnetic field is taken to be negligible and $\underline{E}$ is constant in space. The quantity $C(f F_j)$ denotes the rate of change in $f(\underline{r}, \underline{v}, t)$ as a result of collisions with particles of type j having a distribution $F_j$. The sum $\Sigma_j C(f F_j)$ can be written as

$$\left[\frac{\partial f}{\partial t}\right]_{\text{coll.}} = \Sigma_j C(f F_j) = C(ff) + \underset{\text{ions}}{\Sigma} C(H F_i) + \underset{\text{neutrals}}{\Sigma} C(f F_n).$$

If we assume only binary collisions are of importance, then $C(f F_j)$ is given by

$$C(f F_j) = \int_{V'} \int_r W \sigma_j (\omega,\psi) f(\underline{v}') F_j(\underline{V}') \, d r \, d^3 V'$$

$$- \int_V \int_r W \sigma_j (\omega, \psi) f(\underline{v}) F_j(\underline{V}) \, d r \, d^3 V.$$

The primes denote quantities after scattering. The quantity $W = /\underline{v}'-\underline{v}/$ is the relative speed. The differential cross-section for elastic scattering of the electron through an angle $\psi$ in the center of mass system is $\sigma_j(\omega,\psi)$.

This last expression for $\partial f/\partial t_{\text{coll.}j}$ cannot be simplified further. However, for certain types of external fields and for certain interactions, it has the form

$$\frac{\partial f}{\partial t}_{\text{coll}} = - \frac{f(\underline{r},\underline{v}) - f_o}{\tau(\underline{r},\underline{v})}$$

where $f_o$ is the equilibrium distribution function and $\tau$ is the relaxation time. Its significance is that the rate of approach to equilibrium is

$$(f-f_o)_t = (f-f_o)_{t=0} \; e^{-t/\tau}$$

The relaxation time is of the same order of magnitude as the time between collisions, $1/v$. The problem is immensely simplified when a relaxation time $\tau_i$ exists for $\sum_{\text{ions}} C(f F_i)$. In addition, we ignore the effect of $\underline{E}$ sin $\omega t$ on the heavy ions and assume that $F_j$ has the general form shown in Figure 4 subject to the relation (5).

The next task before us is to incorporate the crucial secondary electron emission process into the formalism. As a first order approximation, one would expect from the previous work that the distribution function

$f(\underline{r},\underline{v},t)$ to have a [redacted] shown in Figure 5. One might examine this part of the problem by placing conditions on $f(o,\underline{v},\phi/\omega)$ and $f(d,\underline{v},\phi_1/\omega)$ in such a manner that the physics of the process is represented properly.

The conductivity [redacted] is a phenomenological constant tensor relating the current to the electric field; namely,

$$J_i = \sum_j [\text{redacted}]_{ij} \epsilon_j,$$

where

$$J_i = q \int [\text{redacted}] v_i f(\underline{v},t)\, d\underline{v}.$$

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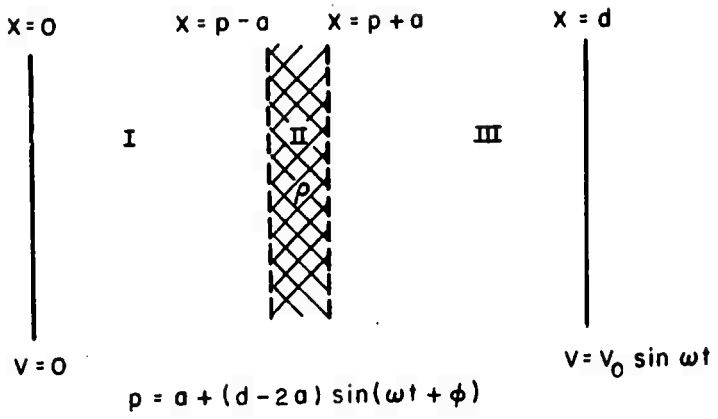


FIG. 1 THE ELECTRON SHEET

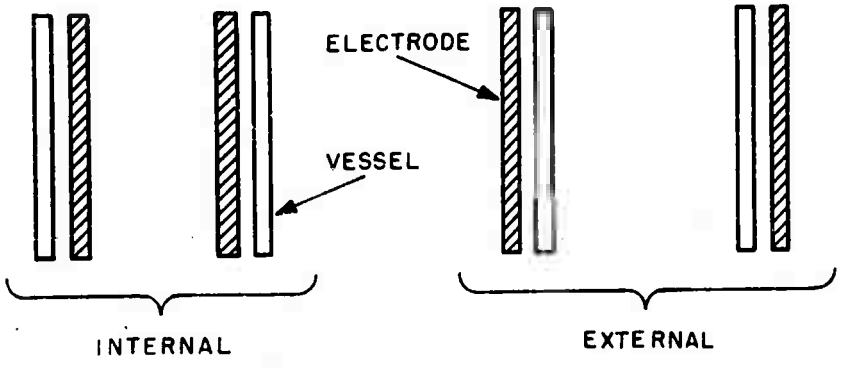


FIG. 2 ELECTRODE PLACEMENT

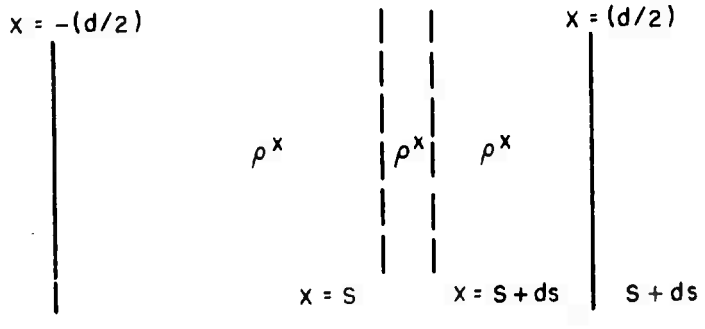


FIG. 3 POSITIVE ION DISTRIBUTION

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6-13-61 Lofgren Graph BENNETT

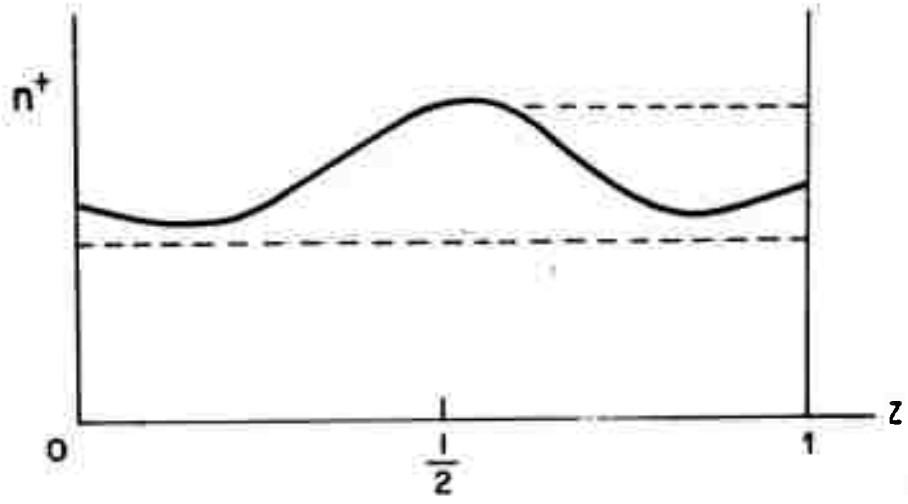


FIG. 4 AVERAGE DISTRIBUTION OF POSITIVE IONS

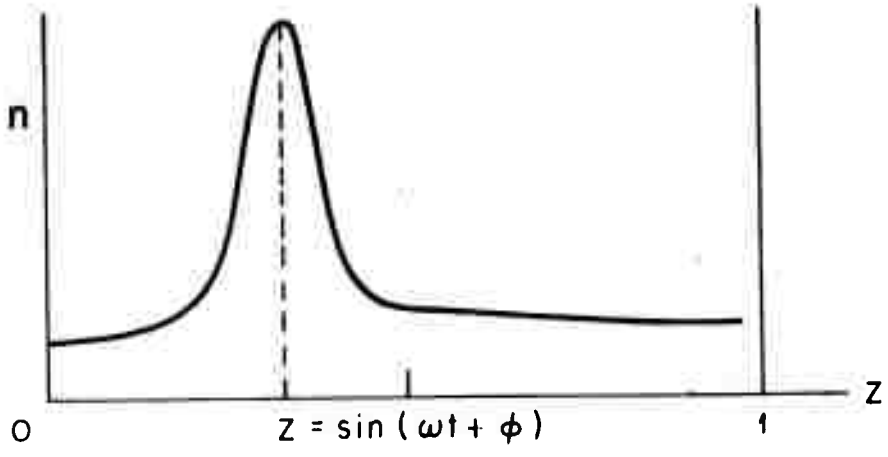


FIG. 5 THE DISTRIBUTION FUNCTION

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